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We study a spherically symmetric fluctuation of scalar dark matter in cosmos and show that it could be the dark matter in galaxies. The local space-time of the fluctuation contains a three dimensional space-like hypersurface with surplus of angle.

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The existence of dark matter in the Universe has been firmly established by astronomical observations at very different length-scales, ranging from single galaxies, to clusters of galaxies, up to cosmological scale (see for example [1]). A large fraction of the mass needed to produce the observed dynamical effects in all these very different systems is not seen. At the galactic scale, the problem is clearly posed: The measurements of rotation curves (tangential velocities of objects) in spiral galaxies show that the coplanar orbital motion of gas in the outer parts of these galaxies keeps a more or less constant velocity up to several luminous radii [2], forming a radii independent curve in the outer parts of the rotational curves profile; a motion which does note corresponds to the one due to the observed matter distribution, hence there must be present some type of dark matter causing the observed motion. The flat profile of the rotational curves is maybe the main feature observed in many galaxies. It is believed that the dark matter in galaxies has an almost spherical distribution which decays like $1/r^2$. With this distribution of some kind of matter it is possible to fit the rotational curves of galaxies quite well [3]. Nevertheless, the main question of the dark matter problem remains; which is the nature of the dark matter in galaxies? The problem is not easy to solve, it is not sufficient to find out an exotic particle which could exist in galaxies in the low energy regime of some theory. It is necessary to show as well, that this particle (baryonic or exotic) distributes in a very similar manner in all these galaxies, and finally, to give some reason for its existence in galaxies.

In previous works it has been explored, with considerable success, the possibility that scalar fields could be the dark matter in spiral galaxies by assuming that the scalar dark matter distributes as an axially symmetric halo [4]. The idea of these works is to explore whether a scalar field can fluctuate along the history of the Universe and thus forming concentrations of scalar field density. If, for example, the scalar field evolves with a scalar field potential $V(\Phi) \sim \Phi^2$, the evolution of this scalar field will be similar to the evolution of a perfect fluid with equation of state $p = 0$, *i.e.*, it would evolve as cold dark matter [5]. However, it is not clear whether a spherical scalar field fluctuation can serve as dark matter in galaxies. In this letter we show that this could be the case. We assume that the halo of a galaxy is a spherical fluctuation of cosmological scalar dark matter and study the consequences for the space-time background at this scale, in order to restrict the state equation corresponding to the dark matter inside the fluctuation. We start from the general spherical symmetric metric and find out the conditions on the metric in order that the test particles in the galaxy possess a flat rotation curve in the region where the scalar field (the dark matter) dominates. Finally we show that a spherical fluctuation of the scalar field could be the dark matter in galaxies.

Assuming thus that the dark matter is scalar, we start with the energy momentum tensor $T_{\mu\nu} = \Phi_{,\mu}\Phi_{,\nu} - 1/2 g_{\mu\nu}\Phi^{,\sigma}\Phi_{,\sigma} + g_{\mu\nu}V(\Phi)$, being Φ the scalar field and $V(\Phi)$ the scalar potential. The Klein-Gordon and Einstein equations respectively are:

$$\Phi^{;\mu}_{;\mu} + \frac{dV}{d\Phi} = 0$$
$$R_{\mu\nu} = \kappa_0[\Phi_{,\mu}\Phi_{,\nu} - g_{\mu\nu}V(\Phi)],$$

where $R_{\mu\nu}$ is the Ricci tensor, $\sqrt{-g}$ the determinant of

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the metric, $\kappa_0 = 8\pi G$ and a semicolon stands for covariant derivative according to the background space-time; $\mu, \nu = 0, 1, 2, 3$.

Assuming that the halo has spherical symmetry and that dragging effects on stars and dust are inappreciable, i.e. the space-time is static, the following line element is the appropriate

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \quad (1)$$

where A and B are arbitrary functions of the coordinate r. Following the analysis made for axisymmetric stationary space-times [6], we consider the Lagrangian for a test particle travelling on the space time described by (1) which is

$$2\mathcal{L} = -B\dot{t}^2 + A\dot{r}^2 + r^2\dot{\theta}^2 + r^2 \sin^2\theta \dot{\varphi}^2 \quad (2)$$

where a dot means derivative with respect to the proper time. From (2) the generalized momenta read:

$$p_t = -E = -B\dot{t} \quad (3)$$
$$p_r = A\dot{r} \quad (4)$$
$$p_\theta = L_\theta = r^2\dot{\theta} \quad (5)$$
$$p_\varphi = L_\varphi = r^2 \sin^2\theta\dot{\varphi} \quad (6)$$

being E the total energy of a test particle and L_i the component of its angular momentum. It can be defined the Hamiltonian $\mathcal{H} = p^\mu \dot{q_\mu} - \mathcal{L}$ and after rescalling the proper time for the lagrangian to equal 1/2 for time-like geodesics, the geodesic equation for material particles (stars and dust) arises

$$\dot{r}^2 - \frac{1}{A}[\frac{E^2}{B} - (L_\theta^2 + \frac{L_\varphi^2}{\sin^2\theta})\frac{1}{r^2} - 1] = 0 \quad (7)$$

We are interested in circular and stable motion of test particles, therefore the following conditions must be satisfied

i) $\dot{r} = 0$, circular trajectories

ii) $\frac{\partial V(r)}{\partial r} = 0$, extreme ones

iii) $\frac{\partial^2 V(r)}{\partial r^2}|_{extr} > 0$, and stable.

being $V(r) = -\left[E^2/B - (L_\theta^2 + L_\varphi^2/\sin^2\theta)/r^2 - 1\right]/A$. Following [7] it is found that the tangential velocity of the test particle is

$$v^{tangential} = v^\varphi = \sqrt{\frac{rB'}{2B}} \quad (8)$$

where $'$ means derivative with respect to r. It is easy to show that if flat rotation curves are required, it arises the following *flat curve condition* from (8), that is $B = B_0 r^l$ with $l = 2(v^\varphi)^2$. With the *flat curve condition*, metric (1) becomes

$$ds^2 = -B_0 r^l dt^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \quad (9)$$

This result is not surprising. Remember that the Newtonian potential ψ is defined as $g_{00} = -exp(2\psi) = -1 - 2\psi - \cdots$. On the other side, the observed rotational curve profile in the dark matter dominated region is such that the rotational velocity v^φ of the stars is constant, the force is then given by $F = -(v^\varphi)^2/r$, which respective Newtonian potential is $\psi = (v^\varphi)^2 \ln(r)$. If we now read the Newtonian potential from the metric (9), we just obtain the same result. Metric (9) is then the metric of the general relativistic version of a matter distribution, which test particles move in constant rotational curves. Function A will be determined by the kind of substance we are supposing the dark matter is made of. Assuming the *flat curve condition* in the scalar dark matter hypothesis, we are in the position to write down the set of field equations. Using (9), the Klein Gordon equation reads

$$\Phi'' + \frac{1}{2r}\left[l + 4 - \frac{A'}{A}r\right]\Phi' + \frac{1}{4}A\frac{dV(\Phi)}{d\Phi} = 0 \quad (10)$$

and the Einstein equations are

$$\frac{A - (l+1)}{r^2} = -\kappa_0\left[\frac{1}{2}\Phi'^2 + AV(\Phi)\right] \quad (11)$$
$$\frac{1}{4r^2}\left[l^2 - \frac{A'}{A}r(l+2)\right] = -\kappa_0\left[\frac{1}{2}\Phi'^2 - AV(\Phi)\right] \quad (12)$$
$$\frac{1}{r^2}\left[1 - A - \frac{A'}{A}r\right] = -\kappa_0\left[\frac{1}{2}\Phi'^2 - AV(\Phi)\right] \quad (13)$$

In order to solve equations (11-13), observe that the combination of the previous equations $1/(2l)[(2-l)$(eq. 11)-4(eq. 12)$+(2+l)$(eq. 13)] implies

$$V = \frac{l}{\kappa_0(2-l)}\frac{1}{r^2} \quad (14)$$

This is a very important result, namely the scalar potential goes always as $1/r^2$ for a spherically symmetric metric with the *flat curve condition.* It is remarkable that this behavior of the stress tensor coincides with the expected behavior of the energy density of the dark matter in a galaxy. It is now possible to solve the field equations, the solution of equations (11-13) implies $A = \left(4 - l^2\right)/\left(4 + C\left(4 - l^2\right)r^{-(l+2)}\right)$, being C an integrations constant. Nevertheless, in this letter we will search a simple solution of the field equations, thus, we look for solutions which stress tensor goes like $1/r^2$. The energy momentum tensor is made essentially of two parts. One is the scalar potential and the other one contains products of the derivatives of the scalar field. Our requirement is possible if $(\Phi_{,r})^2 \sim 1/r^2$ as well, this means that $\Phi \sim \ln(r)$. This implies that the scalar potential is exponential $V \sim \exp(-2\alpha\Phi)$. Using (14) and the ansatz $(\Phi_{,r})^2 \sim 1/r^2$ in (10-13), the solution for the system is

$$A = \frac{4-l^2}{4}, \tag{15}$$

$$\Phi = \sqrt{\frac{l}{\kappa_0}}\ln(r) + \Phi_0, \tag{16}$$

$$V(\Phi) = \frac{l}{2-l}\exp[-2\sqrt{\frac{\kappa_0}{l}}(\Phi-\Phi_0)]. \tag{17}$$

Function A corresponds to an exact solution of the Einstein equations of a spherically symmetric space-time, in which the matter contents is a scalar field with an exponential potential. Let us perform the rescalling $r^2 \to 4r^2/\left(4-l^2\right)$. In this case the three dimensional space corresponds to a *surplus of angle* (analogous to the deficit of angle) one; the metric reads

$$ds^2 = -B_0 r^l dt^2 + dr^2 + \frac{4}{4-l^2}r^2\left[d\theta^2 + \sin^2\theta d\varphi^2\right] \tag{18}$$

for which the two dimensional hypersurface area is $4\pi r^2 \times 4/(4-l^2) = 4\pi r^2/(1-(v^\varphi)^4)$. Observe that if the rotational velocity of the test particles were the speed of light $v^\varphi \to 1$, this area would grow very fast. Nevertheless, for a typical galaxy, the rotational velocities are $v^\varphi \sim 10^{-3}$ $(300km/s)$, in this case the rate of the difference of this hypersurface area and a flat one is $(v^\varphi)^4/(1-(v^\varphi)^4) \sim 10^{-12}$, which is too small to be measured, but sufficient to give the right behavior of the motion of stars in a galaxy.

Let us consider the components of the scalar field as those of a perfect fluid, it is found that the components of the stress-energy tensor have the following form

$$-\rho = T^0{}_0 = \frac{l^2}{(4-l^2)}\frac{1}{r^2} \tag{19}$$

$$P = T^r{}_r = \frac{l(l+4)}{(4-l^2)}\frac{1}{r^2} \tag{20}$$

while the angular pressures are $P_\theta = P_\varphi = -\rho$. The analysis of an axially symmetric perfect fluid in general is given in [6], where a similar result was found (see also [4]).

The effective density (19) depends on the velocities of the stars in the galaxy, $-\rho = (v^\varphi)^4/(1-(v^\varphi)^4) \times 1/(\kappa_0 r^2)$ which for the typical velocities in a galaxy is $-\rho \sim 10^{-12} \times 1/(\kappa_0 r^2)$, while the effective radial pressure is $P = (v^\varphi)^2((v^\varphi)^2+2)/(1-(v^\varphi)^2) \times 1/(\kappa_0 r^2) \sim 10^{-6} \times 1/(\kappa_0 r^2)$, *i.e.*, six orders of magnitude greater than the scalar field density. This is the reason why it is not possible to understand a galaxy with Newtonian dynamics. Newton theory is the limit of the Einstein theory for weak fields, small velocities but also for small pressures (in comparison with densities). A galaxy fulfills the first two conditions, but it has pressures six orders of magnitude bigger than the dark matter density, which is the dominating density in a galaxy. This effective pressure is the responsible for the behavior of the flat rotation curves in the dark matter dominated part of the galaxies.

Metric (18) is not asymptotically flat, it could not be so. An asymptotically flat metric behaves necessarily like a Newtonian potential provoking that the velocity profile somewhere decays, which is not the observed case in galaxies. Nevertheless, the energy density in the halo of the galaxy decays as

$$-\rho \sim \frac{10^{-12}}{\kappa_0 r^2} = \frac{10^{-12}H_0^{-2}}{3r^2}\rho_{crit} \tag{21}$$

where $H_0^{-1} = 3/h\ 10^6 Kpc$ is the Hubble parameter and ρ_{crit} is the critical density of the Universe. This means that after a relative small distance $r_{crit} \sim \sqrt{3/h^2} \approx 3Kpc$ the effective density of the halo is similar as the critical density of the Universe. One expects, of course, that the matter density around a galaxy is smaller than the critical density, say $\rho_{around} \sim 0.06\rho_{crit}$, then $r_{crit} \approx 14Kpc$. Observe also that metric (18) has an almost flat three dimensional space-like hypersurface. The difference between a flat three dimensional hypersurface area and the three dimensional hypersurface area of metric (18) is $\sim 10^{-12}$, this is the reason why the space-time of a galaxies seems to be so flat. We think that this results show that it is possible that the scalar field could be the missing matter (the dark matter) of galaxies and maybe of the Universe.

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